Invariant Hilbert schemes

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The Hilbert scheme is a classical object of algebraic geometry, which plays an important role in the classification of algebraic varieties. It parametrizes the closed subschemes of a prescribed projective space $\mathbf{P}^n$, having a prescribed Hilbert polynomial; in particular, the closed subvarieties of $\mathbf{P}^n$ having prescribed numerical invariants.

In recent years, an analogue of the Hilbert scheme has been constructed in the setting of representations of a reductive group G; they parametrize closed G-stable subschemes X of a prescribed G-module V, such that the coordinate ring of X has a prescribed G-module structure. This "invariant Hilbert scheme" naturally occurs in the study of quotient singularities by finite groups (it often provides a natural desingularization) and in the classification of spherical varieties.

The mini-course will give an introduction to the invariant Hilbert scheme by presenting its construction and basic properties, and discussing examples related to finite groups, spherical varieties, and the $n!$ conjecture.

The prerequisites are familiarity with finite-dimensional representations of reductive groups and with basic notions of affine algebraic geometry. The relevant scheme-theoretic notions will be introduced and discussed during the mini-course.

Some references:

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P. Bravi and S. Cupit-Foutou, *Equivariant deformations of the affine multicone over a flag variety*, Adv. Math. **217** (2008), no. 6, 2800–2821.

M. Haiman and B. Sturmfels, *Multigraded Hilbert schemes*, J. Algebraic Geom. **13** (2004), no. 4, 725–769.

Y. Ito and I. Nakamura, *Hilbert schemes and simple singularities*, in: New trends in algebraic geometry (Warwick, 1996), 151–233, London Math. Soc. Lecture Note Ser. **264**, Cambridge Univ. Press, Cambridge, 1999.

S. Jansou, Déformations des cônes des vecteurs primitifs, *Math. Ann.* **338** (2007), no. 3, 627–667.

S. Kumar and J. F. Thomsen, *A conjectural generalization of the n! result to arbitrary groups*, Transform. Groups 8 (2003), no. 1, 69–94.