MORPHISMS ARE NOT TRIANGULATED

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ABSTRACT. We prove that there is no (pre)triangulated structure on the category of morphisms of a (non-zero) pointed category and other related categories.

Everything comes out after a thorough contemplation of the following well known fact

Every epimorphism in a (pre)triangulated category is split.

Corollary 1. *Let* $\mathcal{A}$ *be a pointed category and* $\mathrm{Mor}(\mathcal{A})$ *the category of morphisms of* $\mathcal{A}$*. If* $\mathcal{A} \neq 0$ *then* $\mathrm{Mor}(\mathcal{A})$ *is not pretriangulated.*

Proof. Let $a \in \mathcal{A}$ be a non-zero object, then

$$\begin{CD} a @>{1}>> a \\ @V{1}VV @VVV \\ a @>>> 0 \end{CD}$$

is a non-split epimorphism in $\mathrm{Mor}(\mathcal{A})$.

Corollary 2. *Let* $\mathcal{A}$ *be a additive category and let* $\mathrm{End}(\mathcal{A})$ *be the category of pairs* (a, α) *where* a *is an object of* $\mathcal{A}$ *and* $\alpha : a \to a$*. If* $\mathcal{A} \neq 0$ *then* $\mathrm{End}(\mathcal{A})$ *is not pretriangulated.*

Proof. Let $a \in \mathcal{A}$ be a non-zero object, then

$$\begin{CD} a \oplus a @>{(1\ 0)}>> a \\ @V{\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}}VV @VV{0}V \\ a \oplus a @>>{(1\ 0)}> a \end{CD}$$

is a non-split epimorphism in $\mathrm{End}(\mathcal{A})$.

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